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## Probing bound states of D-branes\*

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**Abstract** A zero-brane is used to probe non-threshold BPS bound states of  $(p, p+2, p+4)$ -branes. At long distances the stringy calculation agrees with the supergravity calculations. The supergravity description is given, using the interpretation of the  $D = 8$  dyonic membrane as the bound state of a two-brane inside a four-brane. We investigate the short distance structure of these bound states, compute the phase shift of the scattered zero-brane and find the bound states characteristic size. It is found that there should be a supersymmetric solution of type IIA supergravity, describing a bound state of a zero-brane and two orthogonal two-brane, all inside a four-brane, with an additional unbound zero-brane. We comment on the relationship between  $p$ -branes and  $(p-2)$ -branes.

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# 1 Introduction

D-branes [1, 2] have emerged as important objects in string theory. It is thus important to understand various properties of these objects. Properties of bound states at threshold are important tests for string duality conjectures [3, 4]. In this paper we explore, non-threshold BPS <sup>1</sup> bound states between D-branes of dimension  $p$ ,  $p+2$  and  $p+4$ . This is done by probing them with another D-brane. In particular we investigate the short distance structure of these bound states.

D-branes can be used to probe sub-string distances <sup>2</sup>. There is a correspondence between the infra-red world-volume theory and the “space-time” description of the D-brane moving in a background [7]. At sub-string scales, one does not expect to have a space-time description. In fact, one generally does not know an appropriate description of the physics. However, when D-branes are involved one can readily see that the short distance physics is governed by the light modes of the open-superstring [8] (or by the light modes of the closed string together with an infinite tower of massive modes), so a space time description is not available, but the physics is under control. In cases where some of the supersymmetries are not broken another description comes in, namely that of a moduli space of a supersymmetric world-volume theory. If one-quarter of the supersymmetries are un broken, the metric on the moduli space is protected from higher loop corrections. In effect, a space-time description means that low-velocity particles follow geodesics of some metric, but this is exactly what a moduli space means, so even at sub-string scales there are situations where the physics of two objects (or more) can be effectively described by a space-time [8]. However the crucial dependence of this description on the probing agent means that it is not a universal description, and thus might differ from our notion of space-time. Classically, the D-brane are singular at  $r = 0$ . The world-volume description of the region near the D-brane shows that as another D-brane approaches the “singularity”, the physics is described by a transition to another branch, and is not singular.

The string description of the bound states we are going to consider is

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<sup>1</sup>All these states should form an algebra, which may elucidate some underlying structure of string theory [5].

<sup>2</sup>See [6] for a treatment of strings scattered off D-branes.

given by open-superstring ending on the brane with some world-volume gauge fields turn on. As discussed in [9, 10] this endows a  $p$ -brane with an  $RR$  charge of a  $(p-2)$ -brane. Further, these are BPS states [11], thus this string description, is a reasonable description for these bound-states. We compare this description to a supergravity description at long distances and find that they agree.

After preliminaries in section (2), the bound state of a two-brane and a zero-brane  $(2-0)$  is studied in section (3). We compare the string description to a supergravity description by comparing the long-range potential between a zero-brane and the  $(2-0)$  bound state. In section (4) we treat the  $(4-2)$  bound state. We compute the long-range, velocity dependent potential between the bound state and a zero-brane and compare them to a supergravity calculation. We also compute the phase shift the scattered zero-brane acquires after scattering from the bound-state at short distances. From the phase shift we compute the absorption probability of the zero-brane by this bound state and the size of the bound-state. Section (5) is devoted to the study of the  $(4-2-2-0)$  bound state, where the two two-branes are orthogonally embedded in the four-brane. We end with conclusions.

## 2 Probing the $(p, p-2)$ bound state

Starting with the action of a  $p$  brane moving in a background, let us concentrate on the coupling to the  $RR$  sector [12]

$$I_{RR} = T_p \int_{W_{p+1}} Str \ C \wedge e^{2\pi\alpha' F}. \quad (1)$$

Here  $T_p = \sqrt{\pi}(4\pi^2\alpha')^{(3-p)/2}$  [13, 10],  $F = dV - \frac{B}{2\pi\alpha'}$ ,  $V$  is the world volume gauge field and  $B$  is the two-form NS-NS gauge field. A  $p$  brane with a constant magnetic field of the world volume gauge field strength  $F$  will carry a  $RR$  charge depending on the form of  $F$ . Here we are assuming that the  $p$  brane is compactified on a torus. One can think of this as representing the bound state of a  $p$  brane with various lower dimensional branes. We will compare our result to the supergravity description of some of these objects. A constant magnetic field on the  $p$  brane is relatively easy to treat and we will mainly scatter zero-branes off various configuration in order to learn about the property of the bound states. This will be done by computing the one

loop vacuum amplitude for open superstring with the appropriate boundary conditions [13].

The one loop vacuum amplitude is the phase shift of the probe after the scattering [14], and defines a potential  $V(r^2)$  through the equation

$$A(b, v) = - \int d\tau V(r^2 = b^2 + \tau^2 \frac{v^2}{1 - v^2}), \quad (2)$$

where  $b$  is the impact parameter and  $v$  is the velocity of the D-brane probe. The short distance behavior is governed by the light open string modes while the long distance is governed by the light closed string modes. Thus when approximating the integrals one should take care to match those two approximation [8].

Given two identical parallel D-brane with the same condensation  $F$  on their world-volume, it is known that the one loop vacuum amplitude is just the same as when the condensation is zero, except for a multiplicative factor of  $(1 + 2\pi\alpha' f^2)$  [15, 16, 11] ( $f$  is the non zero entry of  $F$ ). This factor expresses the change in the mass of the brane, due to the binding with a lower brane, and the change in the  $RR$  charge. Thus the mass of the bound state of a  $p$ -brane and a  $(p - 2)$ -brane is

$$m_{(p,p-2)}^2 = m_p^2 + m_{p-2}^2, \quad (3)$$

which is the dual to the mass formula in [17]. The brane's original charge is un-modified, but there is an additional charge density of order  $f$  on each brane, which is the  $RR$  charge of the lower dimensional brane. Some D-brane configurations, with world volume gauge field turn on, were considered in [11, 17, 18, 19, 20, 22, 23].

For completeness we give the following. In terms of  $q = e^{-\pi t}$ , we define

$$f_1(q) = q^{1/12} \prod_{n=1} (1 - q^{2n}). \quad (4)$$

$$f_2(q) = \sqrt{2} q^{1/12} \prod_{n=1} (1 + q^{2n}). \quad (5)$$

$$f_3(q) = q^{-1/24} \prod_{n=1} (1 + q^{2n-1}). \quad (6)$$

$$f_4(q) = q^{-1/24} \prod_{n=1} (1 - q^{2n-1}). \quad (7)$$

The limit of  $t \rightarrow 0$  one has,

$$f_1(q) \rightarrow \frac{1}{\sqrt{t}} e^{-\pi/(12t)}. \quad (8)$$

$$f_2(q) \rightarrow e^{\pi/(24t)}(1 - e^{-\pi/t}). \quad (9)$$

$$f_3(q) \rightarrow e^{\pi/(24t)}(1 + e^{-\pi/t}). \quad (10)$$

$$f_4(q) \rightarrow \sqrt{2}e^{-\pi/(12t)}. \quad (11)$$

We will also find it convenient to have the behavior of the  $\Theta(\nu t, it)$  (Jacobi theta functions) in the limit  $t \rightarrow 0$

$$\frac{\Theta_1(i\epsilon t, it)}{\Theta_1'(0, it)} \rightarrow -e^{(\pi\epsilon^2 t)} \frac{t}{i} \frac{\sin(\pi\epsilon)}{\pi}. \quad (12)$$

$$\frac{\Theta_2(i\epsilon t, it)}{\Theta_2(0, it)} \rightarrow e^{(\pi\epsilon^2 t)} (1 + 4\sin^2(\pi\epsilon)e^{-\pi/t}). \quad (13)$$

$$\frac{\Theta_3(i\epsilon t, it)}{\Theta_3(0, it)} \rightarrow e^{(\pi\epsilon^2 t)} (1 - 4\sin^2(\pi\epsilon)e^{-\pi/t}). \quad (14)$$

$$\frac{\Theta_4(i\epsilon t, it)}{\Theta_4(0, it)} \rightarrow e^{(\pi\epsilon^2 t)} \cos(\pi\epsilon). \quad (15)$$

## 2.1 Compact Branes

When some of the space times coordinate are compact their effect on the configuration of the D-branes depends on whether the compact dimensions are an  $NN$ ,  $ND$  or  $DD$  directions. In the case a  $NN$  direction is compact the integral over the momentum in that direction becomes a sum over the allowed momenta. If the compact direction is a  $DD$  direction there is no momentum integral to begin with, however there are infinite number of open string configuration that wrap around the compact direction. Thus the mass of the open string is now  $M^2 = \frac{b^2}{(2\pi\alpha')^2} + \frac{1}{\alpha'} \sum (\text{oscillators}) + (nR/\alpha')^2$ ,  $R$  is the radius of the compact direction, and there is a string configuration for each  $n$ . In the case of a  $ND$  direction there are no momentum integrals and no winding modes, thus there is no change in the one loop computation. This is of course what one expects from T-duality which changes a  $DD$  direction to a  $NN$  direction but the number of  $ND$  directions remain the same.

The one loop amplitude for a configuration of a  $p$  brane and an  $l$  brane moving parallel to each other with one  $NN$  direction compactified is ( $L = 2\pi R$ )

$$A = \frac{C_{l-1}}{2\pi} \int \frac{dt}{t} e^{-(\frac{b^2 t}{2\pi\alpha'})} (8\pi^2 \alpha' t)^{-(\#NN-1)/2} \Theta_3(0, 8i\pi^2 \alpha' t/L^2) B \times J. \quad (16)$$

$B$  and  $J$  are the usual contribution from the bosonic and fermionic oscil-

lators respectively. Similarly for a compactified  $DD$  direction one finds

$$A = \frac{C_l}{2\pi} \int \frac{dt}{t} e^{-(\frac{b^2 t}{2\pi\alpha'})} (8\pi^2 \alpha' t)^{-(\sharp NN)/2} \Theta_3(0, itL^2/2\pi^2 \alpha') B \times J. \quad (17)$$

For example, in the case of  $p = 6, l = 2$  and one of the  $NN$  directions being compact, one can calculate the  $v^2$  term of the potential to be ( $\beta = \frac{r^2 L^2}{16\pi^2 \alpha'^2}$ )

$$V = -\frac{\pi C_1 v^2 L^2 \coth(\sqrt{\beta})}{(8\pi^2 \alpha')^2 \sqrt{\beta}}, \quad (18)$$

from which the moduli space metric can be read off. When  $\beta$  is large the potential falls like  $r^{-1}$  as expected from a six-brane, and when  $\beta$  is small the metric falls like  $r^{-2}$  as expected from a five brane.

From equation (16) the important scale that determines the behavior of a system with  $NN$  compact directions, is  $bL$ . If  $bL$  is large than the system will behave as if it is un-compact and vice versa. So if we probe deeply a system compactified on an  $NN$  direction then it will behave as if the compactification scale is small. Similarly If  $L$  is small but we go far away the system will behave as if it is un-compactified. For a compact  $DD$  direction the relevant scale is of course  $b/L$ .

In the next sections we will have to deal with compact dimensions that are different than  $NN, DD$  or  $ND$ . We will be faced with compact coordinates that satisfy a  $D$  or  $N$  boundary condition on one end of the string and some condensation on the other end, we shall call them  $NF$  and  $DF$  conditions. When an  $NF$  or  $DF$  direction is compact things are different. For a  $DF$  condition there will not be any momenta integral but will be something like a winding, and vice versa for the  $NF$  coordinates. In order to avoid this complication we will always assume that the radius of compactification is large enough as to neglect those effects, even in the large  $r$  limit, and we will treat those directions as if they are un-compactified (from the modes point of view).

### 3 (2-0) bound state

For the two-brane there is only one relevant term in the expansion (1) and it is  $A \wedge F$ , where  $A$  is the  $RR$  gauge field carried by the zero-brane. We will assume that the two-brane is compactified on  $T^2$ . If one chooses

$$F = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & f \\ 0 & -f & 0 \end{pmatrix}$$

then,

$$\int A \wedge F \rightarrow \int f d^2\sigma \int Ad\tau. \quad (19)$$

Requiring  $\int f = 2\pi$  gives the two-brane action a term  $T_0 \int Ad\tau$ , which is the coupling of a zero-brane to a  $RR$  background.

As  $f = \text{const}$ , the zero-brane  $RR$  charge of this configuration is proportional to  $fL^2$  where  $L^2$  is the area of the compactified two-brane.

Let us compute the velocity-dependent potential, between the (2-0) bound state and another zero-brane moving with velocity  $v$ . The one loop amplitude (the phase shift) takes the form ( $\tan(\pi\epsilon) = 2\pi\alpha'f$ ,  $\tanh(\pi\nu) = v$ ),

$$A = \frac{1}{2\pi} \int \frac{dt}{t} e^{-(\frac{b^2 t}{2\pi\alpha'})} B \times J, \quad (20)$$

$$B = \frac{1}{2} f_1^{-6} \Theta_4^{-1}(i\epsilon t) \frac{\Theta_1'(0)}{\Theta_1(\nu t)}, \quad (21)$$

$$\begin{aligned} J = & \left\{ -f_2^6 \frac{\Theta_2(\nu t)}{\Theta_2(0)} \Theta_3(i\epsilon t, it) + f_3^6 \Theta_2(i\epsilon t, it) \frac{\Theta_3(\nu t)}{\Theta_3(0)} \right. \\ & \left. + i f_4^6 \frac{\Theta_4(\nu t)}{\Theta_4(0)} \Theta_1(i\epsilon t) \right\}. \end{aligned} \quad (22)$$

The existence of the  $NS(-1)^F$  sector, the third term in equation (22), is a consequence of the new boundary condition for the open super-string. Instead of having 2  $ND$  coordinates which gives fermionic zero modes in that sector those two coordinates now have different boundary conditions  $FD$  [15],

$$\partial_\sigma X^\mu + 2\pi\alpha' F_\nu^\mu \partial_\tau X^\nu = 0 \quad (\sigma = 0), \quad (23)$$

$$\partial_\tau X^\mu = 0 \quad (\sigma = \pi). \quad (24)$$

Similarly for the fermionic coordinates [16].

We treat the case where the non-zero component of  $F$  are constant. One can solve for the modes and compute the one loop amplitude. A short cut to

the right answer is to start with the expression in [16] which is for the case of an electric field on both branes (in their case a 9-brane). To get a magnetic field one just substitutes  $\epsilon \rightarrow i\epsilon$  and to get a Dirichlet boundary condition on one of the branes one can formally take the condensation on that brane to  $\infty$ . This has the following effect, one substitutes

$$\Theta_1(i\epsilon) \rightarrow i\Theta_4(i\epsilon). \quad (25)$$

$$\Theta_2(i\epsilon) \rightarrow \Theta_3(i\epsilon). \quad (26)$$

$$\Theta_3(i\epsilon) \rightarrow \Theta_2(i\epsilon). \quad (27)$$

$$\Theta_4(i\epsilon) \rightarrow i\Theta_1(i\epsilon). \quad (28)$$

Further more, because of the Dirichlet boundary condition on one end there are no zero-modes in the bosonic sector. The velocity dependence is as in [14], thus we end up with equations (20-22).

Of course, when  $\epsilon$  goes to zero in equations (20-22) one gets back just the expression for a zero-brane scattered off a two-brane.

Let us check that one gets the right charge for the zero-brane inside the two-brane. Taking only the  $RR$  sector (the third term in equation (22)) one finds that the charge per unit volume of the zero-brane inside the two-brane is proportional to  $\sim T_2 \tan(\pi\epsilon) = 2\pi T_2 \alpha' f$  exactly as expected, and the  $RR$  sector has the right sign to represent interaction of two zero-brane of same charge.

One can compute the one loop amplitude in various limits. When the distance between the branes  $r$  is large one gets for the velocity dependent potential

$$V = -\Gamma(5/2) \frac{(2 + 2\sin^2(\pi\epsilon) + 2\sinh^2(\pi\nu) - 4\sin(\pi\epsilon)\cosh(\pi\nu))}{\cos(\pi\epsilon)\sqrt{(8\pi^2\alpha')}} \left(\frac{2\pi\alpha'}{r^2}\right)^{5/2}. \quad (29)$$

Where  $\cosh(\pi\nu) = \frac{1}{\sqrt{(1-v^2)}}$  and  $\sinh(\pi\nu) = \frac{v}{\sqrt{(1-v^2)}}$ .

These results of course hold with some modification to all T-dual configurations. For instance the long range potential between a bound state of a four-brane and a two-brane and another two-brane parallel to the one inside the four-brane is, ( $Q_4$  is the four-brane charge)

$$V_{string} \sim -Q_4 \frac{(2 + 2\sin^2(\pi\epsilon) + 2\sinh^2(\pi\nu) - 4\sin(\pi\epsilon)\cosh(\pi\nu))}{\cos(\pi\epsilon)} r^{-3}. \quad (30)$$



We can compare this result, with the conjectured supergravity description of the bound state of a two-brane inside the four-brane [21]. The supergravity configuration of a two-brane inside a four-brane, was first derived as the  $D = 8$  dyonic membrane. It will be convenient to write down its eleven-dimensional interpretation. The metric takes the form

$$\begin{aligned}
ds_{11}^2 &= (H\tilde{H})^{1/3}[H^{-1}(-dt^2 + dy_1^2 + dy_2^2) + \tilde{H}^{-1}(dy_3^2 + dy_4^2 + dy_5^2) \\
&\quad + dx_1^2 + \cdots dx_5^2]. \\
F_4^{(11)} &= \frac{1}{2}\cos(\zeta)\star dH + \frac{1}{2}\sin(\zeta)dH^{-1}\wedge dt\wedge dy_1\wedge dy_2 \\
&\quad + \frac{3\sin(2\zeta)}{2\tilde{H}^2}dH\wedge dy_3\wedge dy_4\wedge dy_5.
\end{aligned} \tag{31}$$

Here  $H = 1 + \frac{\gamma}{r^3}$ ,  $\tilde{H} = 1 + \frac{\gamma\cos^2(\zeta)}{r^3}$  and  $\star$  is the Hodge dual in  $R^5(x_1\cdots x_5)$ . Now this is the metric of the eleven dimensional two-brane inside a five-brane, but when one considers any of the  $y_i$   $i = 3 - 5$  as the eleventh direction we get a two-brane inside a four-brane. Further the two-brane charge is  $Q_2 \sim \gamma\sin(\zeta)$  and the four-brane charge  $Q_4 \sim \gamma\cos(\zeta)$ . If we choose  $y_5$  as the eleven dimension (so its radius is small), the other two  $y$ 's are also compactified but on a large circle, as discussed in section (2).

Now one can calculate the velocity dependent potential between a two-brane and this bound state, where the two-branes are parallel, using the metric and gauge fields in equation (31). This is easiest done in the static gauge, and one can readily use the formulas in [24] to find,

$$V_{sugra} \sim \frac{\gamma}{r^3}[4\sin(\zeta) + 2\cos^2(\zeta) - 4 - v^2\cos^2(\zeta)] \tag{32}$$

Comparing this to equation (30) we find they do not agree. The reason for that is that while in the supergravity calculation we have worked in the “static gauge” in which the time like parameter of the world volume is equal to  $X_0$ , this is not the case in the string calculation. These expressions then identify potentials in two different reference frames. The string calculation can easily be converted to this frame. Observe that in the string calculation we have taken the expression for the potential of the form

$$A = -\int d\tau V(r^2 = b^2 + \tau^2 \sinh^2(\pi\nu)), \tag{33}$$

so that  $\tau \neq X_0$ . In order to get the string theory answer corresponding to the observer  $\tau = X_0$  one just needs to multiply equation (30) by a factor

$\frac{v}{\sinh(\pi\nu)} = \cosh^{-1}(\pi\nu)$ . Then the two expressions, that of the string theory and that of the supergravity agree to order  $v^2$  when one identifies  $\zeta = \pi\epsilon$ .

When  $r$  becomes very small the appropriate expansion is of  $t \rightarrow \infty$ . We introduce a cutoff  $\Lambda$ , and the  $t$  region  $0 \rightarrow \Lambda$  is governed by the light closed string modes which make a non-singular contribution of order  $\sim r^2$ . Then one gets (for  $v = 0$ )

$$A \approx \int_{\Lambda}^{\infty} \frac{dt}{t} e^{-t(\frac{b^2}{2\pi\alpha'} - \pi/2 + \pi\epsilon)} (8\pi^2 \alpha' t)^{-(1/2)} \quad (34)$$

Now when  $b < \pi/2 - \pi\epsilon$  a tachyon develops in the open string spectrum and the expression becomes complex [25]. However this happens at a slightly smaller distance than in the pure two-brane case.

What happens when  $\epsilon$  grows, the largest it could be is  $\epsilon = 1/2$ . Huresticly as  $\epsilon$  grows one gets more and more “towards” a Dirichlet boundary condition on the two-brane, and the tachyonic instability starts at smaller distances. One can see that the constant term and the  $v^2$  term in the potential goes to zero while the  $v^4$  term grows.

Notice that a bound state configuration of  $(2-0)$  is dual to a bound state of a D-string and an elementary string. In the first case this is described by turning on a magnetic field on the world-volume and in the second it is by turning on an electric field in the world volume [17].

## 4 (4-2) bound state

The world-volume of the four brane is five dimensional and the coupling we are going to consider in this section is  $C \wedge F$ , where  $C$  is the  $RR$  three form gauge field coupled to the two-brane. The four-brane will be wrapped around  $T^2$ . Taking

$$F = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & f & 0 & 0 \\ 0 & -f & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

the  $F \wedge F$  coupling will not contribute, so we will have a bound state of a two-brane and a four-brane. Of course one can orient the two-brane inside the four brane in more than one way, by choosing which elements of  $F$  will be

non zero. The membrane world volume occupies the directions where  $F = 0$ , so if  $F_{12} \neq 0$  then the membrane is in the 3, 4 direction inside the four-brane, and it is the 1, 2 direction which is compact (not the directions the two-brane occupies).

The phase shift for a moving zero-brane probe (with velocity  $v$ ) in the presence of the  $(4 - 2)$  bound state is ( $\tanh(\pi\nu) = v$ ,  $\tan(\pi\epsilon) = 2\pi\alpha'f$ ),

$$A = \frac{1}{2\pi} \int \frac{dt}{t} e^{-(\frac{v^2 t}{2\pi\alpha'})} B \times J. \quad (35)$$

$$B = \frac{1}{2} f_1^{-4} f_4^{-2} \Theta_4^{-1}(i\epsilon t) \frac{\Theta_1'(0)}{\Theta_1(\nu t)}. \quad (36)$$

$$J = \{-f_2^4 f_3^2 \frac{\Theta_2(\nu t)}{\Theta_2(0)} \Theta_3(i\epsilon t, it) + f_3^4 f_2^2 \Theta_2(i\epsilon t, it) \frac{\Theta_3(\nu t)}{\Theta_3(0)}\}. \quad (37)$$

As  $\epsilon \rightarrow 0$  one gets the result for a zero-brane and a pure four-brane. This expression can be evaluated at various limits. Let us first compare the string description with the supergravity description. The appropriate limit is then  $r \rightarrow \infty$ , so  $t \rightarrow 0$ , which is the range when the mass-less closed string modes dominate. One finds a velocity dependent potential,

$$V = -\Gamma(3/2) \frac{(\sin^2(\pi\epsilon) + \sinh^2(\pi\nu))}{\cos(\pi\epsilon) \sqrt{(8\pi^2\alpha')}} \left(\frac{2\pi\alpha'}{r^2}\right)^{3/2}. \quad (38)$$

We turn now to the supergravity description. If there is a zero-brane interacting with the  $(4 - 2)$  bound state, one can compute [26] the velocity dependent potential from null geodesics on the metric (31). Then one finds

$$V_{sugra} \sim \frac{-Q_4}{r^3} \frac{(\frac{v^2}{1-v^2} + \sin^2(\zeta))}{\cos(\zeta)} \quad (39)$$

Equation (39) agrees with equation (38) when one identifies  $\zeta = \pi\epsilon$ <sup>3</sup>. In the string description  $\epsilon = 1/2$  describes an infinite condensation of two-branes on the four-brane. On the supergravity side this is the case  $\zeta = \pi/2$ , which makes the supergravity solution into a pure two-brane. A condensation of infinitely many two-branes on the four-brane have turned it into a two-brane.

Let us now turn to the case that  $r$  is shorter than the string scale. As  $v \rightarrow 0$  the potential between the zero brane and the  $(4 - 2)$  bound state is,

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<sup>3</sup>Notice that we do not have the problem of converting the string calculation to the static gauge, because the supergravity calculation is done differently than in the previous section

$$V = -\frac{\Gamma(-1/2)}{\sqrt{8\pi^2\alpha'}} \left[ \left( \frac{r^2}{2\pi\alpha'} - \pi\epsilon \right)^{1/2} + \left( \frac{r^2}{2\pi\alpha'} + \pi\epsilon \right)^{1/2} - 2\left( \frac{r^2}{2\pi\alpha'} \right)^{1/2} \right] \quad (40)$$

The first two-terms are from the  $NS$  sector of the open string and the last from the  $R$  sector. It exhibits the characteristic of a stretched string potential between the branes. A feature that is due to the probing agent, the zero-brane. The difference in the ground state energy of the different sectors, translates into different effective length for the stretched string. Now if one views  $\epsilon$  as a parameter that can change, then this expression tells as that  $\epsilon$  will want to grow. This means that if we have fixed the charges on the brane, then it is the volume of the compactified brane that will tend to decrease.

To order  $(\pi\epsilon)^2$  The potential becomes

$$V = -\frac{\sqrt{\pi}(\pi\epsilon)^2}{2\sqrt{8\pi^2\alpha'}} \left( \frac{2\pi\alpha'}{r^2} \right)^{3/2} \quad (41)$$

So for small  $r$  and large  $r$  the potential agree to order  $\epsilon^2$ , which will enable us later to approximate some integrals in a simple way. This agreement is a residue of the supersymmetry present when  $\epsilon = 0$ .

At non-zero velocity one can compute the phase shift of the scattered zero-brane. For small  $r$  we can take the  $t$  integral limits from  $0 \rightarrow \infty$  because in this case the  $t \rightarrow 0$  limit of the open string is the same as the closed string.

We consider the case where the velocity ( $v$ ) is small. Then one finds ( $\pi\nu \approx v$ )

$$A = \int \frac{dt}{t} e^{-\left(\frac{b^2 t}{2\pi\alpha'}\right)} \left( \tan\left(\frac{vt}{2}\right) + \frac{\cosh(\pi\epsilon t) - 1}{\sin(vt)} \right) \quad (42)$$

This gives,

$$e^{iA} = \frac{\Gamma\left[\frac{ib^2}{4v\pi\alpha'} + \frac{1}{2} - \frac{i\pi\epsilon}{2v}\right] \Gamma\left[\frac{ib^2}{4v\pi\alpha'} + \frac{1}{2} + \frac{i\pi\epsilon}{2v}\right]}{\Gamma\left[\frac{ib^2}{4v\pi\alpha'} + 1\right] \Gamma\left[\frac{ib^2}{4v\pi\alpha'}\right]}. \quad (43)$$

Equation (42) exhibits a tachyonic instability at  $b^2 < 2\pi^2\alpha'\epsilon$ , this will translate after analytic continuation to a large imaginary part of equation (43), which would mean a very small norm for the scattered wave function (i.e absorption). An incoming zero-brane in a plane wave state with velocity in the  $z$  direction, will be multiplied by the phase shift (equation 43) after scattering, so

$$e^{ikz} \rightarrow e^{ikz + iA(b^2 = x_\perp^2, v)}. \quad (44)$$

The norm of the outgoing wave function is given by,

$$|e^{iA}| = \left[ \frac{\sinh^2(\frac{b^2}{4v\alpha'})}{\cosh^2(\frac{b^2}{4v\alpha'}) + \sinh^2(\frac{\pi^2\epsilon}{2v})} \right]^{1/2}. \quad (45)$$

If the norm of the wave function is much less than 1, it signals the breakdown of the WKB-Eikonal approximation. For low  $v$  this will happen when  $b^2 < 2\pi^2\alpha'\epsilon$ .

If we Fourier transform the outgoing wave we will get the scattering amplitude as a function of the incoming momenta ( $k = \frac{v}{g}$ )

$$f(k, \theta) \sim \exp[-\sqrt{2}k \sin(\theta/2)(\sqrt{\pi\epsilon + (gk)^2} - \pi\epsilon)^{1/2}]. \quad (46)$$

In the limit  $v \gg \epsilon$  we get

$$f(k, \theta) \sim e^{-\sqrt{2}\sin(\theta/2)(kl_p^{11})^{3/2}} \quad (47)$$

as in [8], which shows that the physical scale is the eleven dimensional Planck length  $l_p^{11} = g^{1/3}l_s$  [27]. In the limit  $\epsilon \gg v$  one finds

$$f(k, \theta) \sim e^{-\sin(\theta/2)\frac{g}{\sqrt{\pi\epsilon}}k^2} \quad (48)$$

However in this limit it is easy to see that the approximation is not valid.

Now  $(1 - |e^{iA}|^2)$  is the probability of absorbing a zero-brane by this bound state at impact parameter  $b$ . At very low velocities ( $v \ll \epsilon$ ) one sees that the probability is  $\sim 1$  for  $b/2\pi\alpha' < \pi\epsilon$  and zero otherwise. One can interpret a scale  $r_0$  at which there is a large absorption probability, as giving the effective scale of the bound state. Of course this depends on the probing agent. thus the above result is what we expect from a state with characteristic length scale in string units of  $\sqrt{\pi\epsilon}$ . For very small  $\pi\epsilon$  this gives a characteristic scale for the bound state (as seen by the zero-brane)  $\sim \frac{(2\pi\alpha')}{L}$ .

The probability of absorbing a zero-brane by this bound state when the zero-brane is an incoming state  $e^{ikz}\phi(b)$  is

$$P_{abs} = \int d^4b |\phi(b)|^2 (1 - \|e^{iA}\|^2) \quad (49)$$

If one assumes that  $\pi^2\epsilon \ll v$  and that  $\phi(b)$  is zero outside a region of volume  $V_4$  and constant in it, then one finds

$$P_{abs} \approx \frac{\Omega_3}{2V_4} [(4v\alpha')^2 \ln 2 + (\pi\epsilon)^2 (2\pi\alpha')^2 (\frac{2}{3} \ln 2 - \frac{1}{6})] \quad (50)$$

Where  $\Omega_3$  is the area of the unit three-sphere. The first term in (50) is present for the pure four-brane, where it is interpreted as a signature for resonances [28]. The second term represents the effective size of the bound state.

## 5 Probing the (4-2-2-0) bound state

We take the four-brane to be wrapped around  $T^4$ . If one chooses

$$F = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & f & 0 & 0 \\ 0 & -f & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & f \\ 0 & 0 & 0 & -f & 0 \end{pmatrix}$$

then the four-brane will be endowed with two-brane charge ( two orthogonal two-branes) and zero-brane charge, due to the coupling

$$\frac{1}{2}A \wedge F \wedge F + C \wedge F. \quad (51)$$

The mass of this bound state is

$$m_{4-2-2-0}^2 = m_4^2 + m_2^2 + m_2^2 + m_0^2 \quad (52)$$

The phase shift of a moving zero-brane in the background of this bound state is

$$A = \frac{1}{2\pi} \int \frac{dt}{t} e^{-(\frac{b^2 t}{2\pi\alpha'})} B \times J. \quad (53)$$

$$B = \frac{1}{2} f_1^{-4} \Theta_4^{-2}(i\epsilon t) \frac{\Theta_1'(0)}{\Theta_1(\nu t)}. \quad (54)$$

$$\begin{aligned} J = & \left\{ -f_2^4 \frac{\Theta_2(\nu t)}{\Theta_2(0)} \Theta_3^2(i\epsilon t, it) + f_3^4 \Theta_2^2(i\epsilon t, it) \frac{\Theta_3(\nu t)}{\Theta_3(0)} \right. \\ & \left. + f_4^4 \frac{\Theta_4(\nu t)}{\Theta_4(0)} \Theta_1^2(i\epsilon t) \right\}. \end{aligned} \quad (55)$$

When  $v = 0$  one finds that the one loop amplitude vanishes, due to one of the identities of the Jacobi functions [29]. This means that a configuration of a (4-2-2-0) bound state of this type together with an additional zero-brane is a stationary solution of the supergravity equations, and should preserves some super-symmetries, probably a quarter <sup>4</sup>.

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<sup>4</sup>After writing this work, I have learned that a T-dual description of this configuration was discussed in [22]

In the limit of large  $r$  one finds a potential

$$V = -\Gamma(3/2) \frac{2(1 - \cosh(\pi\nu)) \sin^2(\pi\epsilon) + \sinh^2(\pi\nu)}{\cos^2(\pi\epsilon) \sqrt{(8\pi^2\alpha')}} \left(\frac{2\pi\alpha'}{r^2}\right)^{3/2}. \quad (56)$$

Notice that to order  $v^2$  one gets the same result as in the case of a zero-brane scattered off a four-brane [26].

In the limit of small  $r$  one finds

$$A = \int \frac{dt}{t} e^{-(\frac{b^2 t}{2\pi\alpha'})} \tan(vt/2). \quad (57)$$

Exactly like in the case of the zero-brane and the pure four-brane. One can evaluate the phase shift as in [8],

$$e^{iA} = \frac{\Gamma[\frac{ib^2}{4v\pi\alpha'} + \frac{1}{2}] \Gamma[\frac{ib^2}{4v\pi\alpha'} + \frac{1}{2}]}{\Gamma[\frac{ib^2}{4v\pi\alpha'} + 1] \Gamma[\frac{ib^2}{4v\pi\alpha'}]}. \quad (58)$$

Thus the moduli space is the same as in the case of a zero-brane moving in the background of a four-brane,

$$ds^2 = \frac{1}{g} \left(1 + \frac{g}{2r^3}\right) (dr^2 + d\Omega_4^2). \quad (59)$$

Notice however that although the results are similar to those in the case where there is a zero-brane scattered off a pure four-brane, the physics is different. For instance in the latter case at large distances the physics is governed by gravity alone, while in the  $(4 - 2 - 2 - 0)$  case there are gauge field interactions as well.

If we assume that the condensation on the four brane was not the same in both direction that is

$$F = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & f_1 & 0 & 0 \\ 0 & -f_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & f_2 \\ 0 & 0 & 0 & -f_2 & 0 \end{pmatrix}$$

and  $f_1 \neq f_2$ , then the one loop has the same form as equation(53-55), with the change,  $\Theta_j^2(i\epsilon t) \rightarrow \Theta_j(i\epsilon_1 t) \Theta_j(i\epsilon_2 t)$ . The configuration of this bound state with a zero-brane is not static any more. Assuming we take a configuration of a four-brane two two-brane and a one-brane the different  $f$ 's represent different areas for the two  $T^2$ 's. At short distances this gives a potential

$$V = -\frac{\Gamma(-1/2)}{\sqrt{8\pi^2\alpha'}} \left[ \left( \frac{b^2}{2\pi\alpha'} - \pi(\epsilon_1 - \epsilon_2) \right)^{1/2} + \left( \frac{b^2}{2\pi\alpha'} + \pi(\epsilon_1 - \epsilon_2) \right)^{1/2} - 2 \left( \frac{b^2}{2\pi\alpha'} \right)^{1/2} \right] \quad (60)$$

As in the previous section  $(\epsilon_1 - \epsilon_2)$  will tend to grow. In this case this means that there is a force that will make the area of one of the  $T^2$  larger than the other. So the space time will tend to develop very different scale in some directions.

The phase shift for the scattering of the zero-brane can be computed for small  $r$  and it has the form of equation (43) with the substitution  $\epsilon \rightarrow (\epsilon_1 - \epsilon_2)$ . This is also true for equations (46-50). The characteristic scale of the bound state is  $r_0^2 \sim 2\pi^2\alpha'(\epsilon_1 - \epsilon_2)$ .

## 6 Conclusions

In this paper we give a string description of the bound states of  $(p, p+2, p+4)$  D-branes. We compare the string description with a supergravity description, using the interpretation of the dyonic membrane as the bound state of a two-brane inside a four-brane. Both description agree at large distances. We compute the velocity dependent potential between a zero-brane probe and these bound states and the phase shift of a scattered zero-brane at short distances. The size of the bound state as seen by a zero-brane is estimated by looking at the absorption cross section. We find that the size of the bound state ( $r_0$ ) is related to the scale of the compact dimension ( $L$ ) of the higher brane, as  $r_0 \sim L^{-1}$ . The largest it could be is one half the string length. In a certain range of parameters one finds that the high energy scattering at fixed angle is governed by the eleventh dimensional Planck scale.

We have found that a special  $(4 - 2 - 2 - 0)$  bound state does not exert a force on an additional zero-brane. This is evidence that there should exist a solution of the supergravity equations that corresponds to a  $(4-2-2-0)$  bound state and another zero-brane, that preserves a quarter of the supersymmetries. It will be interesting to find the corresponding super-gravity solution. The moduli space, in this case, turns out to be the same as in the pure four-brane zero-brane case. This is also true for the long range interaction between this bound state and a zero-brane, even though the physics of the pure four-brane zero-brane system looks very different.



If one takes the limit of infinite condensation on one of the branes, we saw it transformed that brane into a  $p - 2$ -brane, this was seen in the string and in the supergravity descriptions. This may suggest that a  $p$ -brane is made out of infinitely many  $(p - 2)$ -branes [9, 30].

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